

K22U 2320

Reg. No.: .....

Name : ..... *2022 (odd sem)*

**V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/  
Improvement) Examination, November 2022  
(2019 Admission Onwards)  
CORE COURSE IN MATHEMATICS**

**5B05MAT : Set Theory, Theory of Equations and Complex Numbers**

Time : 3 Hours

Max. Marks : 48

**PART– A**

Answer **any four** questions from this Part. **Each** question carries **one** mark.

1. Give an example of a countable set.
2. Explain Descartes rule of signs.
3. If  $f(x) = 0$  is an equation of odd degree, then it has at least one                      root.
4. Say true or false. “Zero is a complex number”.
5. Find the conjugate of  $6 - 5i$ .

**PART– B**

Answer **any eight** questions from this Part. **Each** question carries **two** marks.

6. Define a denumerable set, give an example.
7. If  $\alpha, \beta, \gamma$  are the roots of  $2x^3 + x^2 - 2x - 1 = 0$ , find
  - i)  $\alpha + \beta + \gamma$
  - ii)  $\alpha\beta\gamma$
  - iii)  $\alpha\beta + \beta\gamma + \alpha\gamma$ .

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8. Search for rational roots of  $f(x) = 2x^3 - 5x^2 + 5x - 3 = 0$ .
9. Show that  $x^5 - 2x^2 + 7 = 0$  has at least two imaginary roots.
10. Transform the equation  $x^3 - 6x^2 + 5x + 12 = 0$ , into an equation lacking second term.
11. Show that if  $x = 1 + 2i$ , then  $x^2 - 2x + 5 = 0$ .
12. Find the modulus and amplitude of  $\sqrt{3} - i$ .
13. Express  $\frac{1+i}{2+3i}$  in the form of  $X + iY$ .
14. A) The solution of a reciprocal equation of first type depends on that of reciprocal equation of first type and of \_\_\_\_\_ degree.  
B) The solution of a reciprocal equation of first type and of degree  $2m$  depends on that of an equation of degree \_\_\_\_\_.
15. Find the roots of  $2x^3 + 3x^2 - 1 = 0$ .
16. A) Write the standard form of a cubic equation.  
B) What is reciprocal equation?

## PART- C

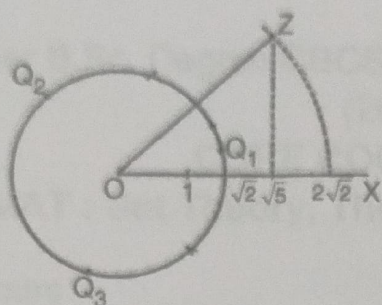
Answer **any four** questions from this Part. **Each** question carries **four** marks.

17. Show that the set  $E_n = \{2n : n \in \mathbb{N}\}$  of even natural numbers is countably infinite.
18. If  $\alpha, \beta, \gamma$  are the roots of  $x^3 + P_1x^2 + P_2x + P_3 = 0$  then find the equation whose roots are  $\alpha^3, \beta^3, \gamma^3$ .
19. Find an upper bound and lower bound for the limit to the roots of  $f(x) = 3x^4 - 61x^3 + 127x^2 + 220x - 520 = 0$ .
20. Solve the reciprocal equation,  $x^4 - 8x^3 + 17x^2 - 8x + 1 = 0$ .





21. Find the points of  $Q_1, Q_2, Q_3$  representing the values of  $\sqrt[3]{z}$  where  $z = \sqrt{5} + i\sqrt{3}$ .



22. A) Define  $n^{\text{th}}$  root of unity.  
B) Define Principal  $n^{\text{th}}$  root of unity.
23. Explain the behaviour of roots of the equation  $ax^3 + 3bx^2 + 3cx + d = 0$ , with respect to discriminant.

#### PART- D

Answer **any two** questions from this Part. **Each** question carries **six** marks.

24. State and prove Cantor's theorem.
25. i) Find the condition that the sum of two roots of  $\alpha, \beta$  of  $x^4 + p_1x^3 + p_2x^2 + p_3x + P_4 = 0$ , may be zero.  
ii) Use the result to find the roots of the equation, whose roots are the six values of  $\frac{1}{2}(\alpha + \beta)$ , where  $\alpha, \beta$  are any roots of  $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$ .
26. If  $\alpha, \beta, \gamma$  are the roots of  $ax^3 + 3bx^2 + 3cx + d = 0$ , then find the equation whose roots are squares of the difference of the roots.
27. Define multiplication and division of two complex numbers.